Reg. No. : 

# Question Paper Code : 91365

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2025

Third Semester

Computer Science and Engineering

MA 3354 — DISCRETE MATHEMATICS

(Common to Computer Science and Engineering (Artificial Intelligence and Machine Learning)/Computer Science and Engineering (Cyber Security)/Computer and Communication Engineering/Artificial Intelligence and Data Science/Computer Science and Business Systems/Information Technology)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — $(10 \times 2 = 20$ marks)

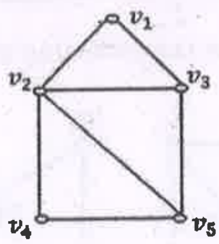
1. Prove that $(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology.

2. Give the converse and the contrapositive of the implication "If it is raining today, then today is a holiday".

3. How many five different letter words can be formed out of the word "ENGINEERING"?

4. State the Pigeonhole principle.

5. Find the complement of the given graph.




6. What is a connected graph? What are the two types of connectedness in digraphs?

7. Prove that every Cyclic group is an abelian group.

8. What is meant by commutative semi group?

9. Draw the Hasse diagram for $(D_{24}, /)$ where $D_{24} = [1, 2, 3, 4, 6, 8, 12, 24\}$.

10. State De Morgan's law in any Boolean Algebra.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that the following premises are inconsistent.

(1) If Jack misses many classes through illness, then he fails high school.

(2) If Jack fails high school, then he is uneducated.

(3) If Jack reads a lot of books, then he is not uneducated.

(4) Jack misses many classes through illness and reads a lot of books. (8)

(ii) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$. (8)

Or

(b) (i) Show that $p \to (q \to r) \Leftrightarrow p \to (\sim q \vee r) \Leftrightarrow (p \wedge q) \to r$. (8)

(ii) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q, Q \to R, P \to M$, and $\sim M$. (8)

12. (a) (i) Use mathematical induction to show that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$, where n is a positive integer. (8)

(ii) Solve the recurrence relation $S(n) - 8S(n-1) - 9S(n-2) = 0$ where $n \geq 2$, and $S(0) = 1, S(1) = 0$ by generating function. (8)

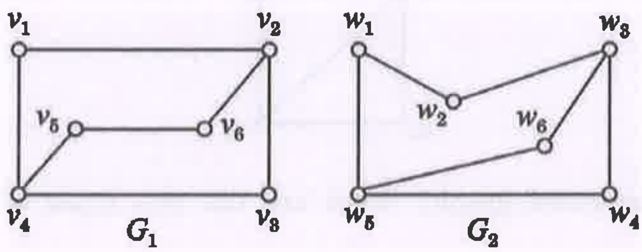
Or

(b) (i) Solve $T(k) - 7T(k-1) + 10T(k-2) = 6 + 8k$, with $T(0) = 1$ and $T(1) = 2$. (8)

(ii) Use the principle of inclusion and exclusion, find the number of integers between 1 to 200 that are not divisible by 2, 3 and 5? (8)

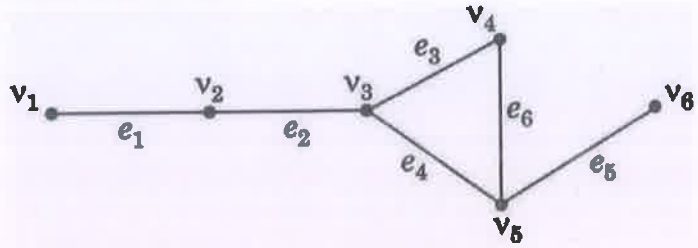
13. (a) (i) Prove that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges. (8)

(ii) Determine whether the following graphs $G_1$ and $G_2$ are isomorphic. (8)




Or

(b) (i) Define incidence matrix of a graph. Write the incidence matrix of the following graph. (8)




(ii) Prove that a given connected graph G is an Euler graph if and only if all the vertices of G are of even degree. (8)

14. (a) (i) Prove that $[Z_6, +_6]$ is an abelian group. (8)

(ii) State and prove Lagrange's theorem. (8)

Or

(b) (i) N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$. (8)

(ii) State and prove the fundamental theorem of group homomorphism. (8)

15. (a) (i) Let $(L, \leq)$ be a lattice in which $\wedge$ and $\vee$ denote the operations of meet and joint respectively. For any $a, b \in L$, prove that $a \leq b \Leftrightarrow a \wedge b = a \Leftrightarrow a \vee b = b$. (8)

(ii) In any Boolean algebra, show that $ab' + bc' + ca' = a'b + b'c + c'a$. (8)

Or

(b) (i) Show that every chain is a distributive lattice. (8)

(ii) In a Boolean algebra prove

(1) $b \leq c \Rightarrow a.b \leq a.c$ and

(2) $b \leq c \Rightarrow a + b \leq a + c$. (8)